Non-anticommutative N=(1,1) Euclidean Superspace*

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Abstract

We study deformations of four-dimensional $N=(1,1)$ Euclidean superspace induced by non-anticommuting fermionic coordinates. We essentially use the harmonic superspace approach and consider nilpotent bi-differential Poisson operators only, which generalizes the recently studied chiral deformation of $N=(\frac{1}{2},\frac{1}{2})$ superspace. We present non-anticommutative Euclidean analogs of $N=2$ Maxwell and hypermultiplet off-shell actions. The talk is based on the paper `hep-th/0308012`.

1. Introduction. This talk reports the results of our recent paper [1] where we discuss nilpotent deformations of $N=(1,1)$ Euclidean superspace.

Deformations of superfield theories are currently a subject of intense study (see, e.g. [2]–[6]). Analogously to noncommutative field theories on bosonic spacetime, non-commutative superfield theories can be formulated in ordinary superspace by multiplying functions given on it via a star product which is generated by some bi-differential operator or Poisson structure P. The latter tells us directly which symmetries of the undeformed (local) field theory are explicitly broken in the deformed (nonlocal) case.

Generic Moyal-type deformations of a superspace are characterized by a constant graded-antisymmetric non(anti)commutativity matrix (C^{AB}). A minimal deformation of Euclidean $N=1$ superspace – more suitably denoted as $N=(\frac{1}{2},\frac{1}{2})$ superspace – was

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considered in a recent paper [7]. For the chiral $N=1$ coordinates $(x_L^m, \theta^\alpha, \bar\theta^{\dot\alpha})$ the noncommutativity was restricted to

$$\theta^\alpha \star \theta^\beta \;=\; \theta^\alpha\,\theta^\beta + \tfrac{1}{2}C^{\alpha\beta}\;,\quad \theta^\alpha \star x_L^m \;=\; \theta^\alpha\,x_L^m\;,\quad x_L^m \star x_L^n \;=\; x_L^m\,x_L^n\;, \tag{1}$$

with $(C^{\alpha\beta})$ being some constant symmetric matrix. Note that the bosonic and the antichiral coordinates have undeformed commutation relations with everyone, so (C^{AB}) is rather degenerate here. For functions A and B of $(x_L^m, \theta^\alpha, \bar\theta^{\dot\alpha})$ the star product (1) is generated as

$$A \star B \;=\; A\,\mathrm{e}^P B \;=\; A\,B + A\,P\,B + \tfrac{1}{2}A\,P^2 B \tag{2}$$

where the bi-differential operator is defined as

$$P \;=\; -\tfrac{1}{2}\overleftarrow{\partial}_\alpha C^{\alpha\beta}\overrightarrow{\partial}_\beta \tag{3}$$

and is nilpotent, $P^3 = 0$. This provides a particular example of a deformed superspace. It retains $N=(\tfrac{1}{2},0)$ of the original $N=(\tfrac{1}{2},\tfrac{1}{2})$ supersymmetry because Q_α commutes with P while $\bar Q_{\dot\alpha}$ does not. It is natural to refer to the deformations generated by a nilpotent Poisson structure like (3) as *nilpotent deformations.*

While the preservation of chirality is the fundamental underlying principle of $N=1$ superfield theories [8], it is the power of *Grassmann harmonic analyticity* which replaces the use of chirality in $N=2$ supersymmetric theories in four dimensions [9, 10]. Therefore, it is natural to look for nilpotent deformations of $N=(1,1)$ Euclidean superspace which preserve this harmonic analyticity (perhaps in parallel with chirality). The basic aim of the present contribution is to describe, following ref. [1], such deformations and to give deformed superfield actions for a few textbook examples of $N=2$ theories. We also discuss the role of the standard conjugation or an alternative pseudoconjugation in Euclidean $N=(1,1)$ supersymmetric theories and their deformations.

Our main novel developments are the analysis of $N=(1,1)$ supersymmetry-breaking deformations in harmonic superspace and the construction of the relevant superfield models. Note that the supersymmetry-preserving deformations of $N=(1,1)$ superspace were considered in [3, 4, 5] (without giving specific dynamical models) and, with making use of the harmonic superspace approach, also in ref. [11].[†] The deformed $N=2$ harmonic superspace and field theory models in it are also addressed in a recent paper [14].

2. Deformations of N=(1,1) superspace in a chiral basis. Our main goal is to generalize the nilpotent deformation of $N=1, D=4$ Euclidean superspace proposed in [7]. In this deformation, one introduces non(anti)commutativity only for one half of the spinor coordinates. By construction, this deformation preserves the chiral representations of $N=1$ supersymmetry. The bi-differential operator of [7] has the form (3) and acts on standard superfields $V(x_L^m, \theta^\alpha, \bar\theta^{\dot\alpha})$.

A a prerequisite, it is appropriate here to note that the (pseudo)conjugation properties of spinors in 4D Euclidean space with the group $Spin(4)=\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ are radically different from those in Minkowski space since left- and right-handed SU(2) spinors

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[†]This paper appeared in hep-th almost simultaneously with [1].

are independent. For $N=1$ Euclidean superspace to have the real dimension (4|4) like its Minkowski counterpart, one is led to apply the following pseudoconjugation of the $SU(2)_L \times SU(2)_R$ spinor Grassmann coordinates (see e.g. [12]): ‡

$$(\theta^\alpha)^* = \varepsilon_{\alpha\beta}\theta^\beta , \qquad (\bar\theta^{\dot\alpha})^* = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta} , \qquad (AB)^* = B^* A^* \qquad (4)$$

where A and B are arbitrary superfields. Here, the map * is a pseudoconjugation which squares to -1 on any odd θ monomial (and on the fermionic component fields) and to $+1$ on any even monomial (and on the bosonic component fields). So, when acting on bosonic fields, it can be identified with the standard complex conjugation. It is straightforward to check that (4) is consistent with the action of the group $Spin(4)$ and preserves its irreducible representations. As an important consequence of the pseudoreality of spinor coordinates, $N=1$ Euclidean chiral superfields can be chosen as real with respect to $*$ (like the general superfields).

Though our main goal is to introduce consistent *nilpotent* deformations of $N=(1,1)$ *harmonic* superspace, it is convenient to start the analysis in the standard $N=(1,1)$ superspace in the *chiral* parmetrization

$$z_L \equiv \left(x_L^m, \theta_k^\alpha, \bar\theta^{\dot\alpha k}\right) . \qquad (5)$$

These coordinates transform under $N=(1,1)$ supersymmetry as

$$\delta_\epsilon x_L^m = 2\mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta_k^\alpha \bar\epsilon^{\dot\alpha k} , \qquad \delta_\epsilon \theta_k^\alpha = \epsilon_k^\alpha , \qquad \delta_\epsilon \bar\theta^{\dot\alpha k} = \bar\epsilon^{\dot\alpha k} , \qquad (6)$$

where ϵ_k^α and $\bar\epsilon^{\dot\alpha k}$ are the transformation parameters. The 'central' bosonic coordinate x^m is related to the 'left' coordinate by

$$x_L^m = x^m + \mathrm{i}(\sigma^m)_{\alpha\dot\alpha}\theta_k^\alpha \bar\theta^{\dot\alpha k} . \qquad (7)$$

As automorphisms we have the Euclidean space spinor group $Spin(4)$ and the R-symmetry group $SU(2) \times O(1,1)$ acting simultaneously on left and right spinors.

Let us dwell in some detail on the (pseudo)conjugation properties of the $N=(1,1)$ superspace. We can assume the Grassmann coordinates to be real with respect to the standard conjugation

$$\widetilde{\theta_k^\alpha} = \varepsilon^{kj}\varepsilon_{\alpha\beta}\theta_j^\beta , \qquad \widetilde{\bar\theta^{\dot\alpha k}} = -\varepsilon_{kj}\varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta j} , \qquad \widetilde{x_L^m} = x_L^m , \qquad \widetilde{AB} = \widetilde{B}\widetilde{A} . \qquad (8)$$

This conjugation squares to identity on any object, and with respect to it the $N=(1,1)$ superspace has the real dimension (4|8). The component spinor fields have the analogous conjugation properties. It is evidently compatible with both $Spin(4)$ and R-symmetries, preserving any irreducible representation of these groups. However, the $N=(\frac{1}{2},\frac{1}{2})$ superspace cannot be treated as a real subspace of the $N=(1,1)$ superspace if one considers only this standard conjugation.

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‡We use the conventions $\varepsilon_{12} = -\varepsilon^{12} = \varepsilon_{\dot1\dot2} = -\varepsilon^{\dot1\dot2} = 1$, $\sigma_m = (i\vec\sigma, I)$ for the basic quantities in the Euclidean space.

3

Surprisingly, in the same Euclidean $N=(1,1)$ superspace one can define an analog of the pseudoconjugation (4)

$$(\theta_k^\alpha)^* = \varepsilon_{\alpha\beta}\theta_k^\beta , \qquad (\bar{\theta}^{\dot\alpha k})^* = \varepsilon_{\dot\alpha\dot\beta}\bar{\theta}^{\dot\beta k} , \qquad (x_L^m)^* = x_L^m , \qquad (AB)^* = B^*A^* \quad (9)$$

with respect to which the $N=(\frac{1}{2},\frac{1}{2})$ superspace forms a real subspace. The existence of this pseudoconjugation does not imply any further restriction on the $N=(1,1)$ superspace. It preserves representations of $N = (1,1)$ supersymmetry and, like (4), is compatible with the action of the group $Spin(4)$. It is also compatible with the R-symmetry group O(1,1). As for the R-symmetry group SU(2), it preserves only some U(1) subgroup of the latter. In other words, the standard conjugation (8) and the pseudoconjugation (9) act differently on the objects transforming by non-trivial representations of this SU(2), e.g. on Grassmann coordinates. The map * squares to -1 on these coordinates and the associated spinor fields, and to $+1$ on any bosonic monomial or field. Only on the singlets of the R-symmetry SU(2), e.g. scalar $N = (1,1)$ superfields and R-invariant differential operators, both maps act in the same way as the standard complex conjugation. In particular, the invariant actions are real with respect to both * and $^\sim$, despite the fact that the component fields may have different properties under these (pseudo)conjugations. Clearly, it is the pseudoconjugation * which is respected by the reduction $N=(1,1) \rightarrow N=(\frac{1}{2},\frac{1}{2})$. Such a reduction preserves the pseudoreality but explicitly breaks the SU(2) R-symmetry.

In chiral coordinates, a *chiral nilpotent deformation* for products of superfields is determined by the following operator,

$$P = -\tfrac{1}{2}\overleftarrow{\partial}_\alpha^k C_{kj}^{\alpha\beta}\overrightarrow{\partial}_\beta^j = -\tfrac{1}{2}\overleftarrow{Q}_\alpha^k C_{kj}^{\alpha\beta}\overrightarrow{Q}_\beta^j \qquad \text{such that}$$

$$APB = -\tfrac{1}{2}(A\overleftarrow{\partial}_\alpha^k)C_{kj}^{\alpha\beta}(\overrightarrow{\partial}_\beta^j B) = -\tfrac{1}{2}(-1)^{p(A)}(\partial_\alpha^k A)C_{kj}^{\alpha\beta}(\partial_\beta^j B)$$

$$= -(-1)^{p(A)p(B)}BPA . \qquad (10)$$

Here, $C_{kj}^{\alpha\beta} = C_{jk}^{\beta\alpha}$ are some constants, $p(A)$ is the supersymmetry Z_2-grading, while $Q_\alpha^k = \partial_\alpha^k$ are the generators of left supersymmetry and the derivatives act as

$$\partial_\alpha^k\theta_i^\beta = \delta_i^k\delta_\alpha^\beta \qquad \text{and} \qquad \bar{\partial}_{\dot\alpha i}\bar{\theta}^{\dot\beta k} = \delta_i^k\delta_{\dot\alpha}^{\dot\beta} . \qquad (11)$$

By definition, the operator P is nilpotent, $P^5 = 0$. It preserves both chirality and anti-chirality and does not touch the $SU(2)_R$. It induces a graded Poisson bracket on superfields [3, 4]. We also demand P to be real, i.e. invariant under some antilinear map in the algebra of superfields. The two possible (pseudo)conjugations introduced above then lead to different conditions

$$(9) \qquad \Longrightarrow \qquad (C_{kj}^{\alpha\beta})^* = C_{\alpha\beta kj} \qquad (12)$$

$$(8) \qquad \Longrightarrow \qquad \widetilde{C_{kj}^{\alpha\beta}} = C_{\alpha\beta}^{kj} . \qquad (13)$$

Since $(APB)^* = B^*PA^*$ and $\widetilde{APB} = \widetilde{B}P\widetilde{A}$, our star-product satisfies the following natural rules:

$$(A \star B)^* = B^* \star A^* , \qquad \widetilde{(A \star B)} = \widetilde{B} \star \widetilde{A} . \qquad (14)$$

4

Under $SU(2)_L \times SU(2)$, the constant deformation matrix C decomposes into a (3,3) and a (1,1) part (see also [4, 6]),

$$C_{kj}^{\alpha\beta} = C_{(kj)}^{(\alpha\beta)} + \varepsilon^{\alpha\beta}\varepsilon_{kj}I \ . \tag{15}$$

It is worth pointing out that the (1,1) part preserves the full $SO(4) \times SU(2)$ symmetry.

Note that the manifestly $N=2$ supersymmetric bi-differential operators of [3, 4] involve flat spinor derivatives D_α^k instead of partial derivatives. Thus they violate chirality. We basically follow the line of [7] and investigate deformations which preserve irreducible representations based on chirality and/or Grassmann harmonic analyticity (see Section 3), but may explicitly break some fraction of supersymmetry.

Given the operator (10), the Moyal product of two superfields reads

$$A \star B = A\,e^P B = AB + APB + \tfrac{1}{2}A\,P^2 B + \tfrac{1}{6}A\,P^3 B + \tfrac{1}{24}A\,P^4 B \tag{16}$$

where the identity $P^5 = 0$ was used. It is easy to see that the chiral-superspace integral of the Moyal product of two superfields is not deformed,

$$\int \mathrm{d}^4x\,\mathrm{d}^4\theta\ A \star B = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\ AB\ , \tag{17}$$

while integrals of star products of three or more superfields are deformed.

In our treatment only free actions preserve all supersymmetries while interactions get deformed and are not invariant under all standard supersymmetry transformations. To exhibit the residual symmetries of a deformed interacting theory, we formulate the invariance condition

$$[K, P] = 0 \tag{18}$$

for the corresponding generators K in the standard $N=(1,1)$ superspace. Clearly, this condition is generically not met by differential operators depending on θ_k^α and the symmetries generated by these are explicitly broken in the deformed superspace integrals. Out of all supersymmetry and automorphism generators, only Q_α^k and $\bar{L}_{\dot\beta}^{\dot\alpha}$ do commute with P of (10). Hence, for a generic choice of the constant matrix $(C_{ik}^{\alpha\beta})$, the breaking pattern is $N=(1,1) \rightarrow N=(1,0)$ for supersymmetry and $SO(4) \times O(1,1) \times SU(2) \rightarrow SU(2)_R$ for Euclidean and R-symmetries.

An exception occurs for the singlet part in (15), i.e. for

$$C_{(kj)}^{(\alpha\beta)} = 0 \quad \Longrightarrow \quad C_{kj}^{\alpha\beta} = \varepsilon^{\alpha\beta}\varepsilon_{kj}I \quad \Longleftrightarrow \quad P_s = -\tfrac{1}{2}\overleftarrow{Q_\alpha^k}\,I\,\overrightarrow{Q_k^\alpha}\ , \tag{19}$$

which is fully $SO(4) \times SU(2)$ invariant and non-degenerate but also fully breaks the right half of supersymmetry.

It is worth noting that it is possible to break less than one half of the supersymmetry if we choose the * conjugation (9). This choice is compatible with the decomposition of $N=(1,1)$ into two $N=(\tfrac{1}{2},\tfrac{1}{2})$ superalgebras, each given by a fixed value for the $SU(2)$ index. Therefore, it allows one to pick a degenerate deformation, e.g.

$$P_{deg}(Q^2) = -\tfrac{1}{2}C_{22}^{12}(\overleftarrow{\bar{Q}_1^2}\,\overrightarrow{Q_2^2} + \overleftarrow{\bar{Q}_2^2}\,\overrightarrow{Q_1^2})\ , \tag{20}$$

5

which does not involve Q^1_α. In this case, only $\bar{Q}_{\dot\alpha 2}$ are broken but not the supercharges $\bar{Q}_{\dot\alpha 1}$. Hence, the deformation P_{deg} preserves $N=(1, \frac{1}{2})$ supersymmetry.

3. Deformations of N=(1,1) harmonic superspace. The basic concepts of the $N=2, D=4$ harmonic superspace [9] are collected in the book [10]. The spinor $SU(2)/U(1)$ harmonics $u^\pm_i$ can be used to construct analytic coordinates $(x^m_A, \theta^{\pm\alpha}, \bar\theta^{\pm\dot\alpha}, u^\pm_i)$ in the Euclidean version of $N=2$ harmonic superspace, that is $N=(1,1)$ harmonic superspace:

$$x^m_A = x^m_L - 2i(\sigma^m)_{\alpha\dot\alpha}\theta^{\alpha k}\bar\theta^{\dot\alpha j}u^-_k u^+_j \,, \qquad \theta^{\alpha\pm} = \theta^{\alpha k}u^\pm_k \,, \qquad \bar\theta^{\pm\dot\alpha} = \bar\theta^{\dot\alpha k}u^\pm_k \quad (21)$$

where $\epsilon^{\pm\alpha}=\epsilon^{\alpha k}u^\pm_k$, $\bar\epsilon^{\pm\dot\alpha}=\bar\epsilon^{\dot\alpha k}u^\pm_k$, and $(x^m_L, \theta^\alpha_k, \bar\theta^{\dot\alpha k})$ are chiral coordinates of $N=(1,1)$ superspace. We extend the (pseudo)conjugations (8) and (9) to the harmonics by

$$\widetilde{u^\pm_k} = u^{\pm k} \qquad \text{and} \qquad (u^\pm_k)^* = u^\pm_k \qquad (22)$$

so that the analytic coordinates are conjugated identically for both choices,

$$\widetilde{x^m_A} = x^m_A \,, \qquad \widetilde{\theta^{\pm\alpha}} = \varepsilon_{\alpha\beta}\theta^{\pm\beta} \,, \qquad \widetilde{\bar\theta^{\pm\dot\alpha}} = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\pm\dot\alpha} \,, \qquad (23)$$

$$(x^m_A)^* = x^m_A \,, \qquad (\theta^{\pm\alpha})^* = \varepsilon_{\alpha\beta}\theta^{\pm\beta} \,, \qquad (\bar\theta^{\pm\dot\alpha})^* = \varepsilon_{\dot\alpha\dot\beta}\bar\theta^{\pm\dot\alpha} \,; \qquad (24)$$

in particular, both square to -1 on spinor coordinates. This means that both maps become pseudoconjugations when applied to the extended set of coordinates. These two pseudoconjugations act identically on invariants and harmonic superfields, e.g. $(A^k B_k)^* = \widetilde{(A^k B_k)}$ or $(q^+)^* = \widetilde{q^+}$, but differ on harmonics or R-spinor component fields, e.g. $(A_k)^* \neq \widetilde{A_k}$. An important invariant pseudoreal subspace is the analytic Euclidean harmonic superspace, parametrized by the coordinates

$$\zeta \equiv (x^m_A \,, \theta^{+\alpha} \,, \bar\theta^{+\dot\alpha}, u^{\pm i}) \,. \qquad (25)$$

The explicit form of supersymmetry-preserving spinor and harmonic derivatives in these coordinates can be found in [10]. The partial derivatives in different bases are related as

$$\partial^L_m = \partial^A_m \,, \qquad D^{++}_L = \partial^{++} = D^{++}_A \,,$$

$$\partial^k_\alpha = -u^{+k}\partial^-_\alpha - u^{-k}\partial^+_\alpha + 2iu^{-k}\bar\theta^{+\dot\alpha}(\sigma^m)_{\alpha\dot\alpha}\partial^A_m \,,$$

$$\bar\partial_{\dot\alpha k} = u^+_k\bar\partial^-_{\dot\alpha} + u^-_k\bar\partial^+_{\dot\alpha} + 2iu^+_k\theta^{-\alpha}(\sigma^m)_{\alpha\dot\alpha}\partial^A_m = u^-_k\bar{D}^+_{\dot\alpha} - u^+_k\bar{D}^-_{\dot\alpha} \qquad (26)$$

where $\partial^\pm_\alpha \equiv \partial/\partial\theta^{\mp\alpha}$, $\bar\partial^\pm_{\dot\alpha} \equiv \partial/\partial\bar\theta^{\mp\dot\alpha}$, $\partial^{++} = u^{+i}\partial/\partial u^{-i}$. A Grassmann analytic superfield is defined by

$$D^+_\alpha\Phi(\zeta, \theta^-, \bar\theta^-, u) = \bar{D}^+_{\dot\alpha}\Phi(\zeta, \theta^-, \bar\theta^-, u) = 0 \qquad (27)$$

and so can be treated as an unconstrained function in the analytic superspace, $\Phi = \Phi(\zeta, u)$.

It is important to realize that the chirality-preserving operator P in (10) also preserves *Grassmann analyticity*. This is seen in the analytic basis using the relations (26),

$$\{\partial^k_\alpha, D^+_\beta\} = \{\partial^k_\alpha, \bar{D}^+_{\dot\beta}\} = 0 \qquad \Longrightarrow \qquad [P, D^+_\beta] = [P, \bar{D}^+_{\dot\beta}] = 0 \,. \qquad (28)$$

For the singlet deformation (19) we have the following deformation operator

$$P_s \;=\; -\mathrm{i}(\sigma^m)^{\alpha\dot\beta}\bar\theta^+_{\dot\beta}\, I\,(\overleftarrow{\partial^A_m}\,\overrightarrow{\partial^-_\alpha} - \overleftarrow{\partial^-_\alpha}\,\overrightarrow{\partial^A_m}) \tag{29}$$

which satisfies $P_s^3 = 0$.

If we do not care about chirality we may add to P any one of the two supersymmetry-preserving operators which in analytic coordinates read

$$L \;=\; \tfrac{1}{2}\big(\overleftarrow{D}^{+\alpha} J \overrightarrow{D}^-_\alpha + \overleftarrow{D}^-_\alpha J \overrightarrow{D}^{+\alpha}\big)\,, \qquad R \;=\; \tfrac{1}{2}\big(\overleftarrow{D}^{+\dot\alpha} \bar J \overrightarrow{D}^-_{\dot\alpha} + \overleftarrow{D}^-_{\dot\alpha} \bar J \overrightarrow{D}^{+\dot\alpha}\big)\,. \tag{30}$$

It is straigtforward to see that these operators indeed do not preserve one of the chiralities (in contrast to the operators P which preserves both ones). They strongly preserve harmonic analyticity, not deforming at all products of analytic superfields $\Phi(\zeta,u)$ and $\Lambda(\zeta,u)$:

$$\Phi\, \mathrm{e}^L \Lambda \;=\; \Phi\,\Lambda\,, \qquad \Phi\, \mathrm{e}^R \Lambda \;=\; \Phi\,\Lambda\,. \tag{31}$$

Note that the superfield geometry of gauge theories in the deformed harmonic superspace with the deformation operator L was studied in [11].

4. Interactions in deformed harmonic superspace. Harmonic superspace with noncommutative bosonic coordinates x^m_A has been discussed in [5]. This deformation yields nonlocal theories but preserves the whole $N=2$ supersymmetry. In contrast, we expect that the deformations of $N=(1,1)$ superspace defined in the previous section will produce much weaker nonlocalities due to their nilpotency. Leaving quantum considerations for future study, we present here the chirally deformed versions of the off-shell actions for some basic theories in harmonic superspace.

We shall limit our attention to the deformation operator P which affects analytic superfields and preserves both analyticity and chiralities, while breaking at least one quarter of $N=(1,1)$ supersymmetry. The free q^+ and ω hypermultiplet actions of ordinary harmonic theory [10] are not deformed in non(anti)commutative superspace:

$$S_0(q^+) \;=\; \int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\, \tilde q^+ D^{++} q^+\,, \qquad S_0(\omega) \;=\; \int \mathrm{d}u\,\mathrm{d}\zeta^{-4}\, (D^{++}\omega)^2\,, \tag{32}$$

where $\mathrm{d}\zeta = \mathrm{d}^4 x_A (D^-)^4$. Non(anti)commutativity arises in interactions, for instance for the self-interactions of the hypermultiplet which contain higher-order terms of the type $\sim \tilde q^+ \star q^+ \star \tilde q^+ \star q^+$. Expanding out the star products yields a finite number of corrections to the local interaction term $(q^+\tilde q^+)^2$.

The interaction of the hypermultiplet q^+ with a U(1) analytic gauge superfield V^{++} can be introduced as in [5], by replacing D^{++} in (32) with the covariant harmonic non-commutative left-derivative,

$$D^{++}q^+ \qquad \Longrightarrow \qquad \nabla^{++}q^+ \;=\; D^{++}q^+ + V^{++} \star q^+. \tag{33}$$

The gauge transformation of the anti-Hermitian V^{++} reads

$$\delta_\lambda V^{++} \;=\; -D^{++}\lambda + [\lambda \overset{\star}{,} V^{++}] \tag{34}$$

where λ is an anti-Hermitian analytic gauge parameter. The generalization to U(n) analytic gauge fields is straightforward. Note again that from the beginning we retain only those symmetries which are unbroken by the deformation of choice.

In Wess-Zumino gauge we have

$$
\begin{aligned}
V_{WZ}^{++} &= (\theta^+)^2\bar{\phi} + (\bar{\theta}^+)^2\phi + 2\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}} \\
&\quad + 4(\theta^+)^2\bar{\theta}^{+\dot{\alpha}}u_k^-\bar{\lambda}_{\dot{\alpha}}^k + 4(\bar{\theta}^+)^2\theta^{+\alpha}u_k^-\lambda_\alpha^k + 3(\theta^+)^2(\bar{\theta}^+)^2u_k^-u_j^-D^{kj} ,
\end{aligned}
\tag{35}
$$

with all components being functions of x_A^m, and a component expansion of the hypermultiplet q^+ which consists of infinitely many terms due to the harmonic dependence. The component expansion of the deformed products is rather complicated since the number of terms increases significantly. E.g. for the singlet deformation P_s, the star product in (33) contains the terms $V^{++}P_s q^+$ and $V^{++}P_s^2 q^+$.

The action for this noncommutative U(1) gauge superfield can be constructed in central coordinates in analogy with the action for commutative $N{=}2$ Yang-Mills theory [13], but it is easier to analyze it in chiral coordinates. Following [13], one constructs the deformed connection for the derivative D^{--} via

$$
D^{++}V^{--} - D^{--}V^{++} + [V^{++} \overset{\star}{,} V^{--}] = 0 , \tag{36}
$$

$$
V^{--}(z_L, u) = \sum_{n=1}^{\infty}(-1)^n \int du_1 \ldots du_n \frac{V^{++}(z_L, u_1) \star V^{++}(z_L, u_2) \ldots \star V^{++}(z_L, u_n)}{(u^+u_1^+)(u_1^+u_2^+)\ldots(u_n^+u^+)} ,
$$

where $(u_1^+u_2^+)^{-1}$ is a harmonic distribution (see [10]). In general, the action for V^{++} contains an infinite number of vertices, with star commutators substituting the ordinary commutators of V^{++} taken from the standard non-Abelian action. The chiral and anti-chiral superfield strengths $\mathcal{W}$ and $\bar{\mathcal{W}}$ in the Euclidean case are independent. They have the form

$$
\mathcal{W} = -\frac{1}{4}(\bar{D}^+)^2V^{--} , \quad \bar{\mathcal{W}} = -\frac{1}{4}(D^+)^2V^{--} , \quad \text{with} \quad \delta_\lambda(\mathcal{W}, \bar{\mathcal{W}}) = [\lambda \overset{\star}{,} (\mathcal{W}, \bar{\mathcal{W}})] , \tag{37}
$$

and satisfy the covariantized chirality and harmonic-independence conditions

$$
\bar{D}_{\dot{\alpha}}^+\mathcal{W} = 0 , \quad \bar{D}_{\dot{\alpha}}^-\mathcal{W} - [\bar{D}_{\dot{\alpha}}^+V^{--} \overset{\star}{,} \mathcal{W}] = 0 , \quad D^{++}\mathcal{W} + [V^{++} \overset{\star}{,} \mathcal{W}] = 0 , \tag{38}
$$

plus analogous conditions on the anti-chiral $\bar{\mathcal{W}}$, as well as $(D^+)^2\mathcal{W} = (\bar{D}^+)^2\bar{\mathcal{W}}$. For the case of the chirality-preserving deformations, one can write down gauge-invariant actions holomorphic in $\mathcal{W}$, such as

$$
S_{\mathcal{W}} \sim \int d^4x_L\, d^4\theta\, (\mathcal{W}^2 + a\mathcal{W} \star \mathcal{W} \star \mathcal{W}), \tag{39}
$$

where a is some constant. It is easy to check that

$$
\delta_\lambda S_{\mathcal{W}} = 0 \quad \text{and} \quad D^{++}S_{\mathcal{W}} = \bar{D}_{\dot{\alpha}k}S_{\mathcal{W}} = 0 . \tag{40}
$$

In the Feynman rules, the only effect of our deformations is a small number of higher-derivative contributions to the standard interaction vertices. Due to the nilpotency of these deformations, the locality of the theory is not jeopardized. It should be straight-forward to evaluate the ensueing mild corrections to the known quantum properties of $N{=}(1,1)$ harmonic superspace.

5. Conclusions. We have considered nilpotent deformations of $N{=}(1,1)$ chiral and Grassmann-analytic harmonic superfields, such that only the anticommutator of half of fermionic coordinates is deformed in a chiral basis. We focussed on those deformations which preserve both chirality and harmonic analyticity, but break $N{=}(1,1)$ supersymmetry: either to $N{=}(1,0)$ or to $N{=}(1,\frac{1}{2})$ (for the degenerate deformation matrix and * conjugation). The second opportunity exists contrary to an assertion of [15]. On the background of non-deformed Euclidean $N{=}(1,1)$ superspace, one can treat such deformations as a soft breaking of the part of supersymmetry and automorphism symmetry. Complete supersymmetry can only be saved at the expense of chirality, though with preserving harmonic analyticity. We gave examples of superfield theories in chiral-nilpotently deformed harmonic superspace. In particular, we have shown how to construct the SO(4) × SU(2) invariant nilpotent deformation of superfield $N{=}(1,1)$ supersymmetric U(1) gauge theory in chiral coordinates.

It would be interesting to understand a possible stringy origin of the deformations considered here and in [11] and to work out the component form of the deformed superfield actions. For the $N{=}(1,1)$ supersymmetry-preserving nilpotent deformation of $N{=}(1,1)$ SYM theory the component action was given in [11]; an analogous consideration for the chirality-preserving SO(4) × SU(2) invariant deformation is now under way [16].

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